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Fibonacci – Protagonist or Witness? Who Taught Catholic Christian Europe about Mediterranean Commercial Arithmetic?*

Abstract: Leonardo Fibonacci (ca. 1170 – after 1240) during his boyhood went to Bejaïa, learned about the Hindu-Arabic numerals there, and continued to collect information about their use during travels to the Arabic world. He then wrote the *Liber abbaci*, which with half a century's delay inspired the creation of Italian abbacus mathematics, later adopted in Catalonia, Provence, Germany etc. Hindu-Arabic numerals, and Arabic mathematics, was thus transmitted through a narrow and unique gate. This piece of conventional wisdom is well known – too well known to be true, indeed. There is no doubt, of course, that Fibonacci learned about Arabic (and Byzantine) commercial arithmetic, and that he presented it in his book. He is thus a witness (with a degree of reliability which has to be determined) of the commercial mathematics thriving in the commercially developed parts of the Mediterranean world. However, much evidence – presented both in his own book, in later Italian abbacus books and in similar writings from the Iberian and the Provençal regions – shows that the *Liber abbaci* did not play a central role in the later adoption. Romance abbacus culture came about in a broad process of interaction with Arabic non-scholarly traditions, at least until ca. 1350 within an open space, apparently concentrated around the Iberian region.

Keywords: Fibonacci, abbacus culture, rule of three, algebra, Ibero-Provençal mathematics

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Benno Artmann in memoriam

1 A Disclaimer

In these times of rampant publicity and rampant legal complaints, it is not uncommon to run into disclaimers explaining in small print what the wonderful product does *not* promise. Let me start by stating, in normal font size however, that the following offers *elements* that have to be incorporated into a synthetic answer, but that are too few and too disparate to allow the construction of this synthesis.

2 Fibonacci's Supposed Role

In the introduction to Leonardo Fibonacci's *Liber abbaci*, we read the following: 'After my father's appointment by his homeland as state official in the customs house of Bugia for the Pisan merchants who thronged to it, he took charge; and, in view of its future usefulness and convenience, had me in my boyhood come to him and there wanted me to devote myself to and be instructed in the study of calculation for some days. There, following my introduction, as a consequence of marvellous instruction in the art, to the nine digits of the Hindus, the knowledge of the art very much appealed to me before all others, and for it I realized that all its aspects were studied in Egypt, Syria, Greece, Sicily, and Provence, with their varying methods; and at these places thereafter, while on business, I pursued my study in depth and learned the give-and-take of disputation.'[1] This can be combined with the prevalent idea about the origin of Italian abbacus mathematics, for instance as expressed recently by Elisabetta Ulivi in her explanation that 'the name 'abbacus school' designates those secondary-level schools that were essentially dedicated to practical arithmetic and geometry and were in the tradition of

1 GRIMM 1976, p. 100. I quote Richard Grimm's translation, based on a critical edition of the introduction for which all the manuscripts that contain it have been used. On the point where all the other known manuscripts differ from the one on which LEONARDUS PISANUS *Liber abbaci* is based (namely whether Fibonacci only speaks of travels to business places or of business travels), Grimm's text is confirmed by Benedetto da Firenze's quotation of the passage (Siena, Biblioteca Comunale degli Intronati, L. IV. 21, ed. ARRIGHI 2004, p. 156).

Leonardo Pisano's *Liber abbaci* and *Practica geometriae*.'[2] Similarly, Warren Van Egmond asserted that all abbacus writings 'can be regarded as ... direct descendants of Leonardo's book.'[3]

If Fibonacci's statement (in particular in the usual reading, where neither 'Greece' nor Provence are noticed) is combined with the opinion expressed by Ulivi and Van Egmond, then it becomes clear that his *Liber abbaci* was the gate through which practical arithmetic was transmitted from the Arabic world to Italy (and from there to the rest of Christian Europe[4]).

In 1997–1998, I discovered that this story is impossible if we look at the specific case of abbacus algebra, but apart from that I followed Descartes' strategy as set forth in the *Discours de la méthode*,[5] to observe the customs and opinions of those among whom I lived until close analysis of the matter would force me to change my observations. When subsequently I *was* forced to do so, I started thinking about the origin of the conventional wisdom.

Part of the explanation is of course that it is easier to look for the lost doorkey within the cone of the street lamp than outside it, in the darkness – to which comes what at another occasion I have called 'the syndrome of the great book', namely 'the conviction that every intellectual current has to descend from a *Great Book* that is *known to us*'.[6]

The only apparently positive evidence comes from a *Livero de l'abbecho* from ca. 1300, which claims in its first line to be 'according to the opinion' of Fibonacci.[7] Close analysis of the treatise[8] shows, however, that this evidence is fallacious. The text moves on two distinct levels, one elementary, the other advanced. The elementary level corresponds to the curriculum of the abbacus school as we know it from two documents.[9] Here we find the rule of three; metrological

2 ULIVI 2004, p. 43. My translation, as everywhere else in the following unless a translator into English is identified either in the text or in the bibliography.

3 VAN EGMOND 1980, p. 7. More examples, also drawn from respected colleagues, are quoted in HØYRUP 2005, p. 24–26; HØYRUP 2007, p. 30 n. 69.

4 Here and everywhere in the paper, 'Christian Europe' refers narrowly to Catholic Christian Europe.

5 DESCARTES *Discours de la méthode*, p. 23.

6 HØYRUP 2003, p. 10.

7 "Quisto ène lo livero de l'abbecho secondo la oppenione de maiestro Leonardo de la chasa degli figluogle Bonaçie da Pisa", *Livero de l'abbecho*, p. 9. The date of the manuscript is discussed in HØYRUP 2005, p. 27 n. 5 and p. 47 n. 37.

8 HØYRUP 2005.

9 One (ARRIGHI 1967) is from the earlier fifteenth, the other (GOLDTHWAITE 1972, p. 421–425 n. 15) from the early sixteenth century; however, nothing suggests the curriculum to have been reduced in the meantime (nor broadened, for that matter).

shortcuts; exchange and barter; elementary alligation; simple interest and elementary composite interest.[10] As can be seen both from the absence of shared problems and from the way mixed numbers are written, this level is fully independent of Fibonacci – except for a misshaped compromise between the normal writing of mixed concrete numbers and Fibonacci's notation for pure mixed numbers.[11] On the other hand, everything on the advanced level is borrowed from the *Liber abbaci* (excepting a final chapter containing mixed sophisticated problems, some of which come from other sources), often demonstrably without understanding.[12] The Fibonacci material thus serves as adornment; it is quite fitting that the copy we possess is a beautiful manuscript on vellum. It follows that the *Liber abbaci* was famous a small century after it was written, and Fibonacci's name a superb embroidered cloak in which the abbacus author in question found it convenient to wrap his book; but also that what this author taught in the abbacus school, and the mathematics he understood, had a different basis.

So far, this concerns a single author (better perhaps, compiler), albeit one of the two earliest abbacus authors whose work has reached us.[13] However, no other abbacus author appears to raise similar claims, except for a fifteenth-century encyclopedia where the claim is even more misleading,[14] and no other author offers material directly copied from the *Liber abbaci*, except for Benedetto da Firenze and a near-contemporary of his, who copy whole sections (the algebra, and chapter 15 part 1, on proportions), but whose own work remains independent of Fibonacci and well within the current abbacus tradition.[15] The situation is slightly

10 The curriculum also encompassed the Hindu-Arabic number system with appurtenant calculation, which Fibonacci is often supposed to have brought to Italy. This is absent from the treatise. I shall not discuss Fibonacci's role in this domain, but just point out that even here there is no positive evidence that his influence was important.

11 On which below n. 37.

12 Fibonacci's composite fractions are understood as normal fractions, which implies that the compiler can never have followed those numerous calculations where they occur. The occasional algebraic "cosa" of which Fibonacci makes use when applying the "regula recta" (first-degree algebra) is either skipped, or the role of this 'thing' as a representative of an unknown quantity is not understood.

13 The other early book is the *Columbia Algorism*, treated below. On the plausible re-dating of the original of this treatise (of which we possess a fourteenth-century copy) to the years 1285–1290, see Høyrup 2007, p. 31 n. 70.

14 Città del Vaticano, Biblioteca Apostolica Vaticana, Ottobon. lat. 3307, which presents itself (fol. 1r) as "Libro di praticha d'arismetrica, cioè fioretti tracti di più libri facti da Lionardo pisano".

15 Benedetto's autograph of his *Praticha d'arismetricha* from 1463 is contained in Siena, Biblioteca Comunale degli Intronati, L. IV. 21, described in detail in Arrighi 2004, p. 129–159; the other 'abbacus encyclopedia', roughly contemporary and anonymous, Firenze, Biblioteca Nazionale Centrale, Fondo Palatino 573, also an autograph, is described in detail ibid., p. 161–195. The evidence that both manuscripts are their respective author's autographs is presented in Høyrup 2010, p. 32 and 39. The claim ibid., p. 39, that the latter manuscript refers to Benedetto's work is due to a misreading, which I use the occasion to correct. It is possible (and even plausible) that both draw their copy of Fibonacci from Antonio de' Mazzinghi's lost *Gran trattato* from the later fourteenth century. But even Antonio's own algebra (as we know it from extracts in the two encyclopediae that were just mentioned) owes nothing to Fibonacci.

different as regards Fibonacci's *Practica geometriae*, insofar as three fourteenth- and fifteenth-century treatises were drawn from it.[16] However, normal abbacus geometries borrow nothing directly (and plausibly very little indirectly) from Fibonacci.

3 Then Whence? Algebra as Initial Evidence

This was the negative part of my argument, which invites a search for alternative gates (or even 'open spaces'). I shall start where my own exploration began, with algebra. None of the two earliest texts contain any algebra – and the compiler of the *Livero de l'abbecho*, as we have seen, does not even know enough about the topic to recognize an algebraic "cosa". The earliest abbacus algebra is contained in the Vatican manuscript of Jacopo da Firenze's *Tractatus algorismi*, written in Montpellier in 1307.[17]

On all accounts – except that it deals with the six fundamental first- and second-degree 'cases' ('equation types'), but then not only with these, and that it uses the term "censo" for the second power – this algebra differs fundamentally not only from Fibonacci's algebra but also from the Latin translations of al-

16 Hughes 2010. Fibonacci's *Practica* is also used so faithfully in Luca Pacioli's *Summa* that misprinted letters in Pacioli's diagrams can often be corrected by means of Boncompagni's edition of the *Practica*: Leonardus Pisanus *Practica geometriae*.

17 Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 4826. The manuscript can be dated by watermarks to ca. 1450. However, stylistic analysis strongly suggests that its algebra belongs together with the rest of the treatise, and that the two manuscripts from which the algebra section is absent are secondary redactions, see Høyrup 2007, p. 5–25. Van Egmond 2009 rejects this conclusion, but with arguments that are refuted by his own earlier publications (and by the sources to which he appeals), see Høyrup 2009. In any case, the Vatican algebra must belong to the earlier fourteenth century. Moreover, other abbacus writings from the earlier fourteenth century share those of its characteristics that enter in the present argument. For our actual purpose, the identity of its author, and even the question whether it is really the earliest abbacus presentation of algebra, are therefore immaterial.

Khwārizmī.[18] It is also very different from Abū Kāmil's algebra, and has only few features in common with al-Karajī's *al-Kāfī fī'l ḥisāb*. Its ultimate root is obviously Arabic algebra. However, its closest Arabic source cannot be any of the erudite treatises that have come down to us; instead, we must think of a mathematical practice in which algebra and commercial calculation were merged.

Moreover, its closest Arabic source cannot be the *immediate* source. Technical works translated directly from the Arabic at the time always contained Arabic loanwords for some of their technical terms; however, no such terminological borrowings are present in the Vatican (or other early abbacus) algebra. The *immediate* source must hence be an environment where algebra was already spoken of in a Romance language. Since Jacopo wrote his treatise in Montpellier (located in Provence, but politically belonging to the Aragon-Catalan kingdom), this environment can reasonably be assumed to have been situated somewhere in the Ibero-Provençal area.

That observation brings to mind Fibonacci's claim to have also learned about the use of the Hindu-Arabic numeral system in Provence, a claim that mostly goes unnoticed. Indeed, if fifteenth-century Provençal mathematics of the abbacus type took its inspiration from Italy, and the Italians had their practice from Fibonacci, what could Fibonacci have learned in Provence? At least one of the premises for this paradox has to be given up.

One early manuscript of the *Liber abbaci*[19] contains another reference to an unexpected locality: the ninth chapter does not simply begin with the words "Incipit capitulum nonum de baractis mercium atque earum similium" as found in Boncompagni's edition[20] but "Hic incipit magister castellanus. Incipit capitulum nonum de baractis mercium atque earum similium". It is difficult to see why a copyist should insert a claim that a certain chapter was copied from a Castilian master if the claim was not in his original; if he did, it would at least show that he knew about such a Castilian treatise and believed to recognize its contents in Fibonacci's text. The passage thus offers evidence of Castilian writing on barter, probably before 1228 (or even 1202, the date of the first version of the *Liber abbaci*, now lost), and in any case before the end of the thirteenth century.

18 Høyrup 2007, p. 147–159.

19 Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1343, fol 47r/v. This manuscript is from the late thirteenth century and thus one of the two oldest manuscripts and older than the one used by Boncompagni for his edition. The passage is already mentioned in Boncompagni 1851–1852, p. 32, but even this piece of disturbing information has been displaced from historians' collective memory.

20 Leonardus Pisanus *Liber abbaci*, p. 118. Henceforth, I shall refer to the *Liber abbaci* by simple page number, always referring to this edition.

Direct evidence for Iberian algebra integrated with commercial arithmetic goes back to the twelfth century – but to Arabic practice. I refer to the *Liber mahamaleth*, a twelfth-century Latin treatise whose title points to Arabic commercial mathematics ("mu'āmalāt"). A systematic presentation of algebra is lost in all manuscripts but repeatedly referred to; since it presents problem types and techniques not dealt with in al-Khwārizmī's algebra, this work cannot be what is spoken of.[21] There are repeated references to Abū Kāmil, who is distinguished from this presentation. However, there are also copious algebraic problems of a kind which we do not find in al-Khwārizmī nor in Abū Kāmil, involving square roots of prices, profits and wages; such problems, though not very common, also turn up in abbacus algebra, starting with Jacopo (assuming that the Vatican algebra is really his).

There is some evidence for further influence from the Maghreb on fourteenth-century developments in abbacus algebra.[22] Firstly, apparent setoffs of the incipient symbolism developed in the Maghreb in the later twelfth century turn up in various Italian manuscripts in the course of the fourteenth century, whereas Jacopo's algebra is totally deprived of symbolism.[23] The scattered character of these setoffs suggests interaction within an open space during the first half of the century, an interaction about which we are however unable to say any more. After the mid-century, Italian abbacus algebra appears to develop largely on its own premises, within its own closed space[24] – developing quite slowly, it must be said.

A *Tratato sopra l'arte della arismetricha*, written in Florence ca. 1390 (Firenze, Biblioteca Nazionale Centrale, Fondo Principale II. V. 152) contains an extensive algebraic section,[25] which suggests a last case of direct or indirect inspiration from Arabic algebra. Firstly, in a wage problem, an unknown amount of money is posited to be a "censo"; Biagio "il vecchio" as quoted by Benedetto da

21 In one place, a problem is solved by means of two unknowns called "res" and "nummus", in a way told to have been taught in the algebra: *Liber mahamaleth*, p. 210 sq. Elsewhere the solution of an indeterminate problem is said to follow a method explained in this same chapter: ibid., p. 427.

22 Since key figures like Ibn al-Yāsamin were active on both sides of the Gibraltar strait, here and elsewhere I use 'Maghreb' in the original sense, indicating the whole Islamic West including al-Andalus.

23 Høyrup 2010, p. 16–25.

24 Or even within a plurality of fairly closed spaces: schemes for calculation with polynomials, though present in some manuscripts before the mid-fourteenth century, are not even mentioned in the Florentine school tradition culminating in Benedetto da Firenze's *Trattato de praticha d'arismetrica* from 1463 and referring back to Biagio il vecchio, Paolo dell'abbaco and Antonio de' Mazzinghi.

25 *Tratato sopra l'arte della arismetricha.*

Firenze[26] had already presented the same problem before ca. 1340, though positing the money to amount to a "cosa". However, the present author does not understand that a "censo" can be a simple amount of money, and therefore feels obliged to find its square root – and then finds the solution as the square on this square root. The author hence cannot himself be familiar with the Arabic meaning of "māl", nor can he however have taken it from Biagio. He uses the terminology without understanding it, and therefore cannot have invented it himself. On the other hand, this rather characteristic problem could not be shared with Biagio if the author's inspiration did not come from the same area – ultimately from the Maghreb, immediately from somewhere in the Romance Ibero-Provençal region.

Another highly plausible borrowing from Maghreb algebra in the same treatise is a scheme for the multiplication of three-term polynomials[27] which emulates the algorithm for multiplying multidigit numbers; the text itself justly refers to the multiplication "a chasella".[28] The 'Jerba manuscript' of Ibn al-Hā'im's *Šarḥ al-Urjūzah al-Yasmīya* does something very similar.[29]

Since these two borrowings occur in the same manuscript and nothing else which I know of from the period suggests any recent contact, interaction through a single gate seems more likely than exchanges in an open space.

I shall say a little about an episode in the reception of Arabic algebra which goes back to the earlier thirteenth century but had negligible impact. Benedetto refers in his own work[30] to a translation made by Guglielmo de Lunis (otherwise known as a translator of Aristotle); Raffaello Canacci is more explicit, and speaks of a translation of "La regola dell'algibra" by Guglielmo "d'arabicho a nostra linghua".[31] In 1521, Francesco Ghaligai copies Canacci,[32] but with reference also to Benedetto; other features of his text confirm that he is familiar with both versions of the story;[33] finally, one manuscript of Gherardo's translation of al-Khwārizmī claims to represent Guglielmo's translation,[34] the existence of which is thus confirmed, even though the ascription itself is obviously wrong.

26 Biagio il vecchio *Chasi exenplari*, p. 89 sq.
27 Earlier manuscripts only present schemes (wholly different in character) for the multiplication of binomials.
28 *Tratato sopra l'arte della arismetricha*, p. 11.
29 Abdeljaouad 2002, p. 33.
30 Benedetto da Firenze *La reghola de algebra amuchabale*, p. 1.
31 Canacci *Ragionamenti d'algebra*, p. 302.
32 Karpinski 1910, p. 209.
33 Høyrup 2008, p. 38.
34 Hughes 1986, p. 223.

It has been proposed that translation into 'our language' should be understood as 'into Latin', and in particular that Guglielmo's translation be identical with the version found in Oxford, Bodleian Library, Ms. Lyell 52.[35] This idea can be safely discarded, since all our evidence apart from the erroneous ascription lists a number of Arabic terms together with Italian explanations; of these terms and explanations there are no traces in the Latin manuscript, which furthermore translates al-Khwārizmī's technical terms differently.

One of the Arabic terms is "elchal", which according to the explanation must stand for "al-qabila". As observed by Ulrich Rebstock (personal communication), the disappearance of the "b" indicates an Ibero-Arabic pronunciation. Apart from this very unspecific confirmation of the role of the Iberian, but Islamic-Iberian, environment, nothing is known about this lost translation – apart from a vague possibility that Fibonacci's occasional use of "avere" instead of "census" in the algebra section of the *Liber abbaci* could be borrowed from Guglielmo.

4 The *Columbia Algorism*

The *Columbia Algorism* is a fourteenth-century copy of a late thirteenth-century original.[36] It is interesting in the present context for several reasons. Firstly, it makes sparse use of a notation for ascending continued fractions, known in Christian Europe primarily from Fibonacci's writings. For instance, Fibonacci would write $\frac{9\ \ 5}{25\ 12}16$ where our notation would be $16+\frac{5}{12}+\frac{9}{12\cdot25}$ (Fibonacci's fractions may be much longer).[37] The *Columbia Algorism* does not write mixed numbers with the fraction to the left, nor does it follow the corrupted usage of the *Livero de l'abbecho*.[38] However, it does use the notation for continued fractions, sometimes

35 Without adopting the thesis, Kaunzner 1985, p. 10–14 gives an adequate survey.
36 New York, Columbia University, Ms. X 511 A13. See also above n. 13.
37 *Columbia Algorism*, p. 155. This is the notation which the *Livero de l'abbecho* (above n. 7 with preceding texts) mixes up with the normal notation for mixed concrete numbers, writing for instance "d. $^{17}/_{49}$ 7 de denaio", "denari $^{17}/_{49}$ 7 of denaro", *Livero de l'abbecho*, p. 18, where his source must have had "7 denari, $^{17}/_{49}$ de denaro" or "denari 7, $^{17}/_{49}$ de denaro". Both the notation for ascending continued fractions and the habit to write the fractional part of a mixed number to the left are borrowings from the Maghreb, where the notations were created in the twelfth century.
38 For instance, in #4 we find "9 e $^{1}/_{2}$" and "8 $^{3}/_{4}$", and in #23 "d 11 e $^{22}/_{25}$ di d", *Columbia Algorism*, p. 33 and 51.

written from right to left (the Maghreb way), sometimes from left to right (an adaptation to the European writing direction).[39] Since nothing else in the treatise points toward Fibonacci and since Fibonacci's continuous fraction line is broken here into two, we may safely assume that he is not the source.

Two other features of the treatise suggest an Iberian connection. Firstly, one of its problems is an atypical use of the dress of a purse. Usually, the purse is found by several persons, which gives rise to a complicated set of linear conditions; what we find in the *Columbia Algorism*[40] is much simpler and analogous to what Fibonacci calls 'tree problems', in accordance with the usual dress for this problem type: 'Somebody had 'denari' in the purse, and we do not know how many. He lost $^{1}/_{3}$ and $^{1}/_{5}$, and 10 'denari' remained for him'.[41] The same problem, only with the unlucky owner of the purse being 'I' and the remaining "dineros" being only 5, is found in the *Libro de arismética que es dicho alguarismo*, a Castilian treatise written in 1393 but based on earlier undated material.[42] Both treatises, moreover, solve the problem by way of a counterfactual question: 'If 7 were 10 ['respectively 5'], what would 15 be?' This is the standard approach of the *Columbia Algorism* as well as the Castilian treatise, but not of other Italian treatises; since the *Columbia Algorism* appears not to have been widely known, the problem type is most likely to have circulated in the Ibero-Provençal area and to have been borrowed from there into the *Columbia Algorism*, even though the opposite passage cannot be excluded.

5 The Rule of Three

The second characteristic feature of the *Columbia Algorism*, on the other hand, can be quite safely attributed to Iberian or at least Ibero-Provençal influence: the way the rule of three is dealt with.[43]

39 In #39 "$^{1}/_{4}\,^{1}/_{2}$" stands for "$^{5}/_{8}$" and "$^{3}/_{4}\,^{1}/_{2}$" for "$^{7}/_{8}$", but in #60 "$^{1}/_{4}\,^{1}/_{2}$" stands for "$^{3}/_{8}$". In #39, moreover, "$^{1}/_{\text{gran}}\,^{1}/_{2}$" stands for "$1^{1}/_{2}$gran", ibid., p. 64 and 81.

40 Ibid., p. 122.

41 Leonardus Pisanus *Liber abbaci*, p. 173.

42 *Libro de arismética que es dicho alguarismo*, p. 167.

43 The 'rule of three' is *a rule*, and to be kept apart from the sort of *problems* (problems of proportionality, 'to a corresponds b, to c corresponds what?') to which it is applied. The rule can be identified through the order of operations to be performed: 'first multiply b and c, then divide by a'. The intermediate result bc has no concrete meaning, whereas the intermediate results of the alternatives (division first) have a concrete interpretation; either 'to 1 corresponds $^{b}/_{a}$' or 'to c corresponds $^{c}/_{a}$ times as much as to a'.

The initial pages of the *Columbia Algorism* are missing; if the rule of three was presented here, we cannot know in which terms this was done. However, most references to the rule within problems are of the kind also encountered in the problem just quoted, through counterfactual questions, 'if a were b, what would c be?' (on the exceptions, see below).

Such counterfactual questions are not absent from other Italian treatises. However, they always occur as secondary examples of the rule of three, after problems confronting two different species of coin, or coin and commodity – or they are found wholly outside the presentation of the rule of three. The *Livero de l'abbecho*, for instance, introduces them separately and at a distance from the rule of three (its very first topic) as a 'rule without a name'.[44] In all Ibero-Provençal treatises I have inspected,[45] however, such counterfactual questions – or related abstract number questions like 'if 4 $^{1}/_{2}$ are worth 7 $^{2}/_{3}$, what are 13 $^{3}/_{4}$ worth?' – always provide the first and basic exemplification of the rule of three.

This observation leaves little doubt about the dependence of the *Columbia Algorism* on an Iberian (or Ibero-Provençal) model, since standard Arabic treatises introduce the rule in a wholly different way. However, the rule of three has much more to say about our topic.

The earliest statement of the rule is found in the *Vedāṅgajyotiṣa*,[46] cautiously to be dated to ca. 400 BCE.[47] In Kuppanna Sastry's translation as quoted by Sarma, this version of the rule runs 'The known result is to be multiplied by the quantity for which the result is wanted, and divided by the quantity for which the known result is given.' The reference to 'the result that is wanted' has some similarity to what we find in the abbacus books – for instance, in Jacopo's *Tractatus algorismi*,[48] 'If some computation should be given to us in which three things were proposed, then we should always multiply the thing that we want to know against

44 *Livero de l'abbecho*, p. 14.

45 That is, beyond the just-mentioned *Libro de arismética que es dicho alguarismo*, in chronological order: the *Pamiers algorism* from c. 1430 (description and excerpts in Sesiano 1984a); the anonymous mid-fifteenth Franco-Provençal *Traicté de la pratique*: *Traicté de la pratique*; Barthélemy de Romans' slightly later, equally Franco-Provençal *Compendy de la praticque des nombres*: Barthélemy de Romans *Compendy*; Francesc Santcliment's Catalan *Summa de l'art d'aritmètica* from 1482: Francesc Santcliment *Summa*; and Francés Pellos' *Compendion de l'abaco* from 1492: Francés Pellos *Compendion de l'abaco*. The *Pamiers algorism*, the *Traicté* and the *Compendy* are connected, but the others are independent of each other and of this group.

46 Sarma 2002, p. 135.

47 Pingree 1978, p. 536.

48 From Høyrup 2007, p. 236 sq., with correction of an error ('in the third thing' instead of 'in the other thing').

that which is not similar, and divide in the third thing, that is, in the other that remains.' This was, with negligible variations, the standard formulation of the rule of three of all abbacus treatises from the *Livero de l'abbecho* onward.

It is not clear from Sarma's quotation but unlikely from the context of his discussion whether already the *Vedāṅgajyotiṣa* refers to a '[rule of] three things', but so do at least Āryabhaṭa, Brahmagupta, Mahāvīra and Bhāskara II.[49] All of them also refer to that which is wanted ("icchā"). Āryabhaṭa's formulation is 'in the (rule of) three magnitudes, after one has multiplied the magnitude 'phala' ['fruit', 'outcome'] with the magnitude 'icchā', the intermediate outcome is divided by the 'pramāṇa' ['measure'].' Here, there is no reference to what is similar/not similar. However, this turns up as secondary information in the formulations of Brahmagupta, Mahāvīra and Bhāskara II – but in ways so different that direct descent can be excluded.[50]

The earliest reference to the rule in an extant Arabic work is in al-Khwārizmī's *Algebra*. Al-Khwārizmī speaks of four quantities, not three. For the rest, interpreters differ on the meaning of his words. For four quantities in proportion "$\frac{a}{b} = \frac{c}{d}$", Frederic Rosen takes al-Khwārizmī to claim that 'a is inversely proportionate to d, and b to d',[51] while Roshdi Rashed states that 'a is not proportional to d' (etc.).[52] A slightly later passage states according to Rosen that among the three known quantities, two 'must necessarily be inversely proportionate the one to the other', according to Rashed that there are two numbers, each of which is not proportionate to its associate; in both cases, these two numbers have to be multiplied. None of this makes much sense mathematically,[53] and the Latin translations of Robert

49 Āryabhaṭa *Āryabhaṭīya*, p. 140; Brahmagupta *Gaṇitādhyāya*, p. 283; Mahāvīra *Gaṇita-sāra-sangraha*, p. 86; Bhāskara II *Līlāvatī*, p. 33.

50 In his *Āryabhaṭīyabhāṣya*, Bhāskara I presents a slightly different explanation, not when commenting upon the text but in connection with his own first example and the linear arrangement of the three known terms: This has been stated, 'in order to bring about a Rule of Three the wise should know that in the dispositions the two similar ('sadṛśa') (quantities) are at the beginning and the end. The dissimilar quantity ('asadṛśa') is in the middle', Bhāskara I *Āryabha-ṭīyabhāṣya*, p. 109 sq. Since it 'has been stated', Bhāskara seems to quote an earlier commentary, see Sarma 2002, p. 137. The phrase 'the wise should know' probably implies this to be *new* knowledge for 'the wise' (the Sanskrit scholars?): when 'the wise' appear elsewhere in the work, they *know*, Bhāskara I *Āryabhaṭīyabhāṣya*, p. 71 and 109.

51 al-Khwārizmī *Algebra* 1831, p. 68.

52 al-Khwārizmī *Algebra* 2007, p. 196.

53 We may presume that both translators have drawn from their familiarity with Euclidean proportion theory, without asking themselves whether al-Khwārizmī would be likely to use the same resource.

of Ketton[54] and Gherardo da Cremona[55] are indeed different while agreeing with each other. Both interpret the essential adjective as 'opposite'.[56] As long as we restrict ourselves to the first statement, this 'opposition' could refer to a graphical scheme.[57] The second passage, however, leaves only one possibility, that the term "mubāyn", translated 'différent' by Mohamed Souissi[58] with reference to exactly this passage, means 'dissimilar', in exact agreement with the secondary explanations of the Sanskrit mathematicians from Brahmagupta onwards.

Most Arabic treatments of the rule have as their primary examples problems confronting commodity and price, and designate the four terms accordingly. They also often present the rule after a short introduction of the proportion concept and the rule of cross multiplication. Sometimes proportions and rule of three are linked, sometimes they are not – and often a formulation including the similar/non similar is involved.

Al-Karajī's *al-Kāfī fī'l ḥisāb* does not link the rule with the preceding presentation of the proportion. His rule runs as follows:[59] 'You find the unknown magnitude by multiplying one of the known magnitudes, for instance the sum or the quantity, by that which is not similar to it, namely the measure or the price, and dividing the outcome by the magnitude which is of the same kind.' Ibn al-Bannā' integrates proportions and the rule of three, and gives the rule in this shape:[60] 'You multiply the isolated given number, (that is, the one which is) dissimilar from the two others, by the one whose counterpart one does not know, and divide by the third known number.' Ibn Ṭabāt also integrates proportions and rule of three, and first gives rules based on the former. Then comes this rule, almost identical with the Italian abbacus formulation:[61] 'The fundament for all 'mu'ā-malāt'-computation is that you multiply a given magnitude by one which is not of the same kind, and divide the outcome by the one which is of the same kind.'

Ibn Ṭabāt was active in Baghdad in the earlier thirteenth century, and a legal scholar rather than a 'mathematician' or 'astronomer-mathematician'. That *his* words should have reached the abbacus school is not credible. We must assume that they reflect the formulation used by merchants in a wide area (apart of course

54 Robertus Kettensis *Liber algebre et almuchabolae*, p. 64.

55 Gerhardus Cremonensis *Liber de algebra et almuchabala*, p. 255.

56 Rozenfeld 1983, p. 45 also agrees, and translates 'protiv'.

57 *Our* scheme, and the scheme used in twelfth-century Toledo, see n. 62; al-Khwārizmī has nothing of the kind.

58 Souissi 1968, p. 96.

59 al-Karajī *al-Kāfī fī'l ḥisāb* 2, p. 16 sq.

60 Ibn al-Bannā' *Talkhīṣ a'māl al-ḥisāb*, p. 88.

61 Ibn Ṭabāt *Ġunyat al-ḥussāb*, p. 43–45.

from the passage 'fundament for all 'mu'āmalāt'-computation', which in the commercial milieu went by itself). If Fibonacci had been taught for more than 'some days' in Bejaïa he might even have encountered it there; in any case, the Italian formulation cannot have been adopted from Fibonacci,[62] nor probably from any other specific 'gate', but by way of participation in a shared open space. The reference of Italian abbacus authors as well as Sanskrit mathematicians to a 'rule of three' suggests that this open space encompassed not only the shores of the Mediterranean but also those of the Arabian Sea.

The origin of the Iberian recourse to counterfactual questions is more enigmatic. It could of course represent a local development; the abstract number question is not difficult to produce by simple abstraction, al-Khwārizmī's example 'ten for eight, how much for four' is not very different; nor would the step from the merely abstract to the explicitly counterfactual be more difficult to make in the Iberian world than elsewhere.

However, there is some reason to believe that at least the abstract formulation circulated in the Arabic commercial world. As it turns out, al-Khwārizmī's 'ten for eight ...' is found in Rosen's, Rashed's and Robert of Ketton's translations – but Gherardo has concrete numbers, 'ten 'cafficii' for six dragmas ...'.[63] The abstract formulation may thus very well have crept into the manuscript tradition after al-Khwārizmī's time. Moreover, Ibn al-Ḫiḍr al-Qurašī, a little-known mid-eleventh-century author from Damascus, explains that the foundation for 'sale and purchase' is the seventh book of Euclid, and then goes on that 'this corresponds to your formulation, 'so much, which is known, for so much, which is known; how much is the price for so much, which is also known?"[64]

62 Fibonacci, when introducing the rule (Leonardus Pisanus *Liber abbaci*, p. 83 sq.) does not speak of a 'rule of three things', as done by the Sanskrit as well as later Italian authors but as common among Arabic mathematicians of 'four proportional numbers, of which three are known but the last unknown'; his rule prescribes the inscription of the numbers on a rectangular "tabula", represented in the treatise by a rectangular frame. This method was also known to the compiler of the *Liber mahamaleth*, and thus in twelfth-century Toledo. It is likely to have inspired Robert's and Gherardo's understanding of "mubāyn" as 'opposite'.

63 al-Khwārizmī *Algebra* 1831, p. 68; al-Khwārizmī *Algebra* 2007 p. 198; Robertus Kettensis *Liber algebre et almuchabolae*, p. 64; Gerhardus Cremonensis *Liber de algebra et almuchabala*, p. 256. This informs us about three manuscripts: the main Arabic manuscript Oxford, Bodleian Library, Hunt 214, used by Rosen and Rashed, and the two lost manuscripts used by Robert and Gherardo. Since Rashed's critical apparatus is incomplete, it is not possible to know how the other Arabic manuscripts look: Rashed has some references to Gherardo's edition, but omits some of its variants on this point; what else he may omit is a guess.

64 Ibn al-Ḫiḍr al-Qurašī *at-Tadhkira bi-uṣūl al-ḥisāb wa l'farā'iḍ*, p. 64.

At this point we may return to the alternative formulation of the rule in the *Columbia Algorism*, first enunciated in general terms: 'Remember, that you cannot state any computation where you do not mention three things; and it is fitting that one of these things must be mentioned by name two times; remember also that the first of the things that is mentioned two times by name must be the divisor, and the other two things must be multiplied together',[65] illustrated by an example and used a couple of times later on.[66] The 'name' that is mentioned twice is obviously a reference to the 'similar' things, so somehow this formulation also points back to the similar and the dissimilar. It was not adopted widely in the abbacus environment, but it must have survived as an undercurrent: Pacioli offers it as an alternative in the Perugia manuscript from 1478,[67] and in almost the same words in the *Summa de arithmetica* from 1494.[68] Also in 1478, moreover, Pietro Paolo Muscharello mixes it into the standard formulation.[69]

Muscharello's treatise was written in Nola, otherwise best known as the birthplace of Giordano Bruno. Nola is located in Campania, an area outside the core of abbacus culture (which flourished between Genova, Milan and Venice to the North and Umbria to the South) and in close contact with the Iberian world. Interestingly, Muscharello also uses the counterfactual structure as his first example of the rule of three. His particular ways suggest that we should not consider Italy of his times as a single community.

Because of the possibility to identify specific markers in the formulations of the rule of three, scrutiny of a larger number of Sanskrit, Arabic and Christian-European presentations of the rule would probably yield more information about points of contact, transmission roads and communities.

65 *Columbia Algorism*, p. 39 sq.

66 Ibid., p. 48 and 50.

67 Luca Pacioli *Tractatus*, p. 19 sq.

68 Luca Pacioli *Summa*, fol. 57r.

69 'This is the rule of 3, which is the fundament for all commercial computations. And in order to find the divisor, always look for the similar thing, which is mentioned twice, and one of these will be the divisor, and I say that it will be the one which is not your request, and this your request you will get by multiplying with the other not similar thing, and this multiplication [i.e. 'product'] you will have to divide by your divisor, and from it will come that which you will require', Pietro Paolo Muscharello *Algorismus*, p. 59.

6 Set Phrases, Abbacus Culture, and Fibonacci

Whoever has read in a few abbacus books will be familiar with phrases like these: 'make this computation for me', 'this is its rule', 'now say thus', 'and it is done, and thus one makes the similar computations' or 'make similar computations thus'. They all point to a teaching concentrated on the solution of problems serving as paradigmatic examples, and they will only have made sense within an institution similar in that respect to the abbacus school. In the *Livero de l'abbecho*, they are particularly copious in those problems that are not taken from Fibonacci, but some of them are glued onto Fibonacci problems without being present in the original.

Fibonacci himself mostly avoids these characteristic locutions; in general, he tries to emulate the style of 'philosophical' mathematics; similarly, he often tries to reformulate the mathematical substance "magistraliter", 'in the way of [school] masters' – this word is found several times in his work.[70] Nonetheless, an occasional 'make similar computations thus' *can* be found in the *Liber abbaci*.

The appearances of the set phrases in Fibonacci's works are by far too few to have inspired their ubiquitous presence in abbacus writings. We may conclude that Fibonacci was so immersed in the style that later unfolds in the abbacus books that he did not manage to eliminate it completely.

In some cases, he distinguishes his own style from what 'we are used to do' or from what is done "vulgariter". An example of the former distinction is in his exposition of the simple false position, taught by means of a tree, of which $\frac{1}{4}$ + $\frac{1}{3}$ are below the ground, which is said to correspond to 21 palms.[71] He searches for a number in which the fractions can be found (taking 12 as the obvious choice), and next argues that the tree has to be divided in 12 parts, 7 of which must amount to 21 palms, etc. But then he explains that there is another method 'which we use' ("quo utimur"), namely to posit that the tree be 12. He concludes that 'therefore it is customary to say, for 12, which I posit, 7 result; what shall I posit so that 21 result?', and finds the solution by the rule of three. This corresponds exactly to what can be found in abbacus books – for instance, in the *Columbia Algorism*: 'The $\frac{1}{3}$ and the $\frac{1}{5}$ of a tree is below the ground, and above 12 cubits appear. ... If you want to know how long the whole tree is, then we should find a number in which $\frac{1}{3}$ $\frac{1}{5}$ is found, which is found in 3 times 5, that is, in 15. Calculate that the whole tree is 15 cubits long. And remove $\frac{1}{3}$ and $\frac{1}{5}$ of 15, and 7 remain, and say thus: 7 should be 12, what would 15 be?'[72]

"Vulgariter, per modum vulgarem ..." are used at least four times[73] to characterize simple stepwise calculation as opposed to a single combined operation by means, e.g., of composite proportions. This would probably be what a practical reckoner preferred. Once addition of $\frac{1}{3}$ and $\frac{1}{4}$ "secundum vulgi modum" is made by taking both fractions of a convenient number ("in casu 12"), similarly a method easily understood by reckoners without theoretical training.[74] More informative is what we find further on.[75] After having found the fourth proportional to 3–5–6 as $(5 \cdot 6)/3$, Fibonacci says that the same question is proposed 'in our vernacular' ("ex usu nostri vulgaris") as 'if 3 were 5, what would then 6 be?' Next, he asks for the number to which 11 has the same ratio as 5 to 9, and restates the question "secundum modum vulgarem" as 'if 5 were 9, what would 11 be?' This tells us that the vernacular practice in which Fibonacci participates – vide his repeated first person plural, 'we use', 'our vernacular' – is of the Ibero-Provençal kind, not similar in its approach to the rule of three to what is later found in Italy. Actually, Santcliment introduces the presentation of the rule of three by saying 'and this species begins in our vernacular, 'if so much is worth so much, how much is so much worth".[76]

Fibonacci is certainly no abbacus author, his scope as well as his ambition goes much beyond that. But as we see, he knew that mathematical culture of which the Italian abbacus school became the most famous representative. His book, furthermore, informs us about how this culture looked at a moment in which it had not yet reached Italy[77] – though not very specifically. It is up to us to try to sort out what comes from which place.

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70 LEONARDUS PISANUS *Liber abbaci*, p. 163, 215 and 364. Ibid., p. 127 instead he refers to the scholarly way as being "secundum artem".
71 Ibid., p. 173 sq.

72 *Columbia Algorism*, p. 79.
73 LEONARDUS PISANUS *Liber abbaci*, p. 115, 127, 204 and 364.
74 Ibid., p. 63.
75 Ibid., p. 170.
76 "E comença la dita specia en nostre vulgar si tant val tant: que valra tant", FRANCESC SANTCLIMENT *Summa*, p. 163.
77 That it had not yet reached Italy is illustrated by yet another reference in the book to vernacular methods (the last one, if I am not mistaken), namely LEONARDUS PISANUS *Liber abbaci*, p. 114. Here the Pisa method to find the profit corresponding to each "libra" invested in a commercial partnership (certainly a real-life method, since it starts by removing as some kind of tax or as payment for the shipping one fourth of the profit) is confronted with calculation "secundum vulgarem modum", which turns out to be the usual partnership rule. A number of apparent Italianisms in the text ("baracta"; "viadium" or "viagium"; "pesones", the Latinized plural of "peso"; "avere"; and various names of goods) might be taken to suggest an Italian background to the

7 Byzantium

As an example of what may perhaps be dug out by careful analysis, I shall mention the role of Byzantium in this landscape of sciences. As quoted, Fibonacci tells us in the introduction to his *Liber abbaci* that one of the places where he encountered study of the art of 'the nine digits of the Hindus ... with their varying methods' was Greece, that is, Byzantium. On several occasions, moreover, he states that a particular problem was presented to him by a Byzantine master;[78] finally, a number of problems tell stories taking place in Constantinople.[79] Of the former group, all examples but one state prices in "bizantii",[80] and all the latter deal with the same Byzantine currency. We may infer that the metrologies occurring in the book, even in wholly artificial problems, were as a rule not chosen at random but thought of in connection with the location where they were in use. Since most problems do not specify where they are supposed to take place,[81] nor where Fibonacci was confronted with them,[82] the metrologies and currencies are likely to carry otherwise hidden information of one or the other kind – or both.

The *Liber abbaci* shares with later books in the abbacus tradition another kind of likely indirect information about the role of Byzantium. Regularly, they start alloying problems with a phrase 'I have silver/gold' of this and this fineness.

Fibonacci uses the structure a few times. It stands in a reference to 'our' (vernacular) way of expressing ourselves.[83] Then it stands as what 'you' should say when stating a problem about alloying of silver.[84] Finally, the locution is used to indicate that an alligation problem dealing with grain is equivalent to one about

Liber abbaci. However, apart from the possibility that Fibonacci – a Tuscan speaker – might introduce such loanwords on his own, it should be noticed that all may just as well come from the Catalan of his epoch.

78 Leonardus Pisanus *Liber abbaci*, p. 188, 190 and 249.

79 Ibid., p. 161, 203, 274 and 296.

80 The one deals with unspecific 'money' ("denarii"), ibid., p. 190.

81 "Bizantii", for example, occur ibid., p. 21, 83 sq., 93–96, 102 sq., 107–109, 113, 115, 119–121, 126, 131, 137 sq., 159–163, 170, 178, 180 sq., 203–207, 223–225, 228–258, 266, 273–277, 279, 281, 283, 310, 313–318, 323, 327–329, 334 sq., 347–349, 396 and 401. Not all of these passages are of course relevant for Byzantium; "bizantii" were also minted in Arabic and crusader countries, see Travaini 2003, p. 245.

82 Actually, I am fairly sure that there are no specified references to locations for such confrontations other than Byzantium, in spite of the open-ended reference to 'the give-and-take of disputation' of the introduction.

83 'When we say, I have bullion at some ounces, say at 2, we understand that one pound of it contains 2 ounces of silver', Leonardus Pisanus *Liber abbaci*, p. 143.

84 Ibid., p. 156.

the alloying of silver.[85] Obviously, Fibonacci sees the 'I have'-structure as characteristic for such problems.

In Jacopo's *Tractatus algorismi*, *all* alloying problems start with 'I have'; the locution is also used in one problem about exchange of money, and in one about money in two purses.[86] All alloying problems in the non-Fibonacci part of the *Livero de l'abbecho* do as much. Later, we find the same opening for instance in Paolo Gherardi's *Libro di ragioni* from 1328;[87] in a *Libro de molte ragioni d'abacho* from c. 1330;[88] in Giovanni de' Danti's *Tractato de l'algorismo* from ca. 1370;[89] in a *Libro di conti e mercatanzie* probably from ca. 1390;[90] in Francesco Bartoli's *Memoriale*, a private notebook written in Avignon before 1425 and containing excerpts from earlier abbacus works;[91] in Piero della Francesca's abbacus treatise;[92] and (with the slight variation, also known by Piero della Francesca,[93] "Io me trovo ...") in Luca Pacioli's *Summa*.[94] It is also found in a Castilian merchant handbook *De arismetica*,[95] and it survives in Christoff Rudolff's *Coss* from 1525.[96]

This distribution of the opening 'I have ...' seems to point to an origin in a particular environment, distinct from that where abbacus problems in general were circulating (a money-dealers' environment, it would seem).

In one Byzantine treatise of abbacus-type (*Ψηφηφορικὰ ζητήματα καὶ προβλήματα*, 'Calculation Questions and Problems') from the early fourteenth century, the first person singular serves not only for alloying problems but also for other problem types (mostly but far from always dealing with possession of or payment in gold coin).[97] If this characterized Byzantine practical mathematics in

85 Ibid., p. 163.

86 Høyrup 2007, p. 125.

87 Paolo Gherardi *Libro di ragioni*, p. 29–31 and 89. In the first of these passages, the first person only initiates problems about gold, whereas a silver problem starts 'There is somebody who has ...'.

88 *Libro de molte ragioni d'abacho*, p. 95–106. Alternating with the formula 'A man has ...'.

89 Giovanni de' Danti *Tractato de l'algorismo*, p. 50–52.

90 *Libro di conti e mercatanzie*, p. 72–74. Gold problems only. Problems about silver are neutral or start 'Somebody has ...'.

91 Francesco Bartoli *Memoriale*, p. 134 sq. Ten instances of 'I have' regarding gold as well as silver, and a single of 'Somebody has' (about gold).

92 Piero della Francesca *Trattato d'abaco*, p. 56–59.

93 Ibid., p. 74.

94 Luca Pacioli *Summa*, fol. 184r–185v.

95 *De arismetica*, p. 45.

96 Christoff Rudolff *Coss*, p. 201 sq. and 215 sq.

97 *Ψηφηφορικὰ ζητήματα καὶ προβλήματα*, p. 21–27.

general, it would be tempting to believe that the Italian and Iberian way to formulate alloying problems had its roots in a *Byzantine* money-dealers environment.[98]

8 Absence of Hebrew Influence

A similar argument can be used to *rule out* another possible line of influence. In Roman Law, it was customary to represent participants in paradigmatic cases by the names Gaius and Titius and less often Maevius.[99] The habit became so familiar in medieval Italy that 'some guy' is spoken of in modern Italian as "un tizio". Similarly, the Babylonian Talmud sometimes (less pervasively) uses Jacob's oldest sons Reuben, Simeon, Levi and Judah for this purpose.[100] Medieval Hebrew practical mathematics[101] took over this usage and applied it much more systematically.[102]

With a single exception, however, abbacus and related Ibero-Provençal writings I know of never adopted the stylistic scheme; the parallel of the 'I have'-opening of alloying problems shows that they would certainly have done so if they had borrowed from the Hebrew tradition.

The single exception is, once again, Muscharello's *Algorismus*. In three problems dealing with the settling of accounts,[103] the protagonists are, respectively,

98 Admittedly, a Byzantine treatise from the next century shows no trace of the style, see HUNGER/VOGEL 1963. On the other hand, the older treatise is local Byzantine in its choice of coins referred to, whereas the younger one is heavily influenced by Italian treatises in this respect, see SCHOLZ 2001, p. 102. It may therefore not say much about Byzantine habits in the twelfth and thirteenth centuries.

99 A search for "Titius" in the electronic version of *Corpus iuris civilis* finds more than 1860 appearances. Often, of course, the name occurs repeatedly within the discussion of a single case – this is exactly why it is useful to have names to refer to. Nonetheless, the omnipresence of this fictive person is impressing.

100 For instance, in *Yebamoth*, fol. 28v Reuben and Simeon have married two sisters, and Levi and Judah two strangers. In *Baḇa Ḳamma*, fol. 8v Reuben sells all his lands to Simeon, who then sells one of the fields to Levi; none of this has anything to do with Genesis.

101 Represented by Ibn 'Ezra's twelfth-century *Sefer ha-mispar*, written in Lucca or Rome ca. 1146: IBN 'EZRA *Sefer ha-mispar*, see SELA 2001, p. 96; the problem collection accompanying Levi ben Geršom's *Sefer maaseh hoshev*: LEVI BEN GERŠOM *Sefer maaseh hoshev*; and Elia Misrachi's *Sefer ha-mispar* (early sixteenth-century): ELIA MISRACHI *Sefer ha-mispar*.

102 Some of ben Geršom's examples, however, deal with anonymous 'travellers', 'merchants' etc., as usual in "mu'āmalāt"- and abbacus texts.

103 PIETRO PAOLO MUSCHARELLO *Algorismus*, p. 154–158 and 193.

Piero and Martino, Rinaldi and Simoni, and Roberto and Martino, and in one about four gamblers, these are Piero, Martino, Antonio and Francisco. Whether this exception is really a borrowing or an independent creation cannot be decided, in particular because German cossic writings begin at the same time to use capital letters for the same purpose – first in Magister Wolack's Latin university lecture about abbacus mathematics from 1467/1468,[104] later occasionally in Rudolff's *Coss* from 1525,[105] and probably by others in between (in this case, inspiration from university teaching of Aristotelian logic is possible). There was thus a need for a way to identify the actors of a problem beyond the traditional 'the first', 'the second', etc., and Muscharello's use of names may have been a self-invented way to meet this need.

9 A Pessimistic Conclusion

In my initial disclaimer I promised that 'the following offers elements that have to be incorporated into a synthetic answer, but that are too few and too disparate to allow the construction of this synthesis'. I am afraid that in particular the negative second part of this pessimistic pledge has been respected. I am also afraid that further research may dig out more elements that have to go into the answer, while making it even more difficult to produce a convincing synthesis – too much of the process has taken place in oral interaction and left no permanent traces. The only reason the Italian situation in itself is *somewhat* better documented is that the Italian merchant class was the effective ruling class of its cities, and eventually even nobility; for these merchant-patricians, mathematics books were objects of prestige – *sacred* objects, almost as the sword was a sacred object for other nobilities. For the ruling classes or culturally hegemonic strata of other areas of importance for our process they were not. The Italian books were therefore conserved with much higher probability than similar books elsewhere; and even in Italy, as one discovers at any attempt to trace *development* – for instance, the development of incipient symbolism – the holes predominate, and the cheese turns out to be all too scarce to satisfy our appetite.

104 WAPPLER 1900, p. 53 sq.

105 CHRISTOFF RUDOLFF *Coss*, p. 211.

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